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ANALYSIS OF WELL PRESSURE DRAWDOWN DATA INTERPRETATION WITH THE PRESSURE DERIVATIVE

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ABSTRACT

The principle of well test data interpretation is presented for measured pressure as response to constant terminal rate in exploratory well drawdown in reservoirs. The purpose of well test interpretation is to fit pressure – time data to some known analytical models, and thus to identify the system's parameters, knowing only the input and output signals, and some other reservoir characteristics, such as boundaries, initial conditions and the shape of drainage area. The decision on stimulating a well is usually arrived at by analyzing the pressure transient region of the test. The objective of this paper is to use some available field data to demonstrate virtues of diagnostic plots in pattern recognition and identification of appropriate interpretation models. The visual pattern recognition also can identify pattern from incomplete and distorted data and so eliminate needs for extensive data preparation and smoothing. We discovered that the interpretation can be improved under user defined mode of diagnostic derivative plot to identify an interpretation model for visual recognition of various flow regimes for reservoir system architecture under consideration. Therefore the question of how do we interpret well test data is answered in general.

Keywords: Well Test, Derivative and Flowing Pressure, Pattern Recognition, Regression

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INTRODUCTION

Obvious problem definition is usually why do we recommend well testing? The response, in colloquial, is to obtain reservoir permeability and skin. Well tests are used for evaluating reservoir characteristics and numerous models of reservoir architectures have been developed. Using source and Green's functions, Adewole and Olafuyi (2010) identified thirty-two different models for both normal and laterally inverted S-shape two layered reservoir architecture intercepted by horizontal wells and concluded that the derivative plots always exhibit a slope of $2\pi h_D$ or $\frac{2\pi h_D \lambda}{x_{ED}}$. Earlier, Jelmert (1991) developed a derivative type curve for horizontal and stimulated wells which was presented at Port Harcourt NAICE conference. His type curves are based on superposition of P_D functions, the recommended superposition function depends on the particular combination of flow periods that have impact on the buildup behavior. This solves the problems of identifying possible linear or bilinear flow period, which may be difficult from conventional type or pressure derivative type curves. Tiab (1988) type curves are useful in well test data analysis. For many years, the only model adopted has been radial flow geometry and have region of validity to interpreting long time or so called middle term data. They were not suited for description of early term data, distorted by wellbore storage effect. The interpretation method is by semi-log plotting analysis of plotting pressure vs. time on this graph.

Well test interpretation is an inverse problem and must be solved by regression technique once the identification of a particular region is follows. In 1970, Ramey (1970), the use of pressure derivatives were introduced to supplement the so called conventional method of semilog graphing and thus brought up the log-log graphing of pressure vs. time. The technique involves matching the logarithm of pressure against the log of time with theoretical type curve and provides very useful quantitative and qualitative information for analyzing both the early well bore storage distorted data, skin effect and reservoir conductivity. Some authors have revealed that this technique be used in case of emergency or as checking device in case the conventional method may fail. So the practical efficiency depends on the specific type-curve that is used. Gringarten (1979) developed a type curve for analysing early-time transient data for various wellbore configurations such as fracture reservoir system.

METHODOLOGY

The ei-function solution describing transient pressure solution to diffusivity equation for homogeneous reservoir with constant terminal rate is given by

$$P_D(t_D) = -\frac{1}{2}E_i\left(-\frac{1}{4t_D}\right) \tag{1}$$

Bourdet and Ayoub (1989) identified the problem influence of variable rate at the opening of the well due to well bore storage and support the use of computer-aided approach which is to extend the mere identification of a straight line on pressure time plot convenient for hand analysis to embrace powerful computing opportunity afforded by computing facilities. Furthermore, the conventional straight line approach fails to use all of the data available and

can result in significant error. The application of derivative considers the response as a whole, from very early-time data to the last recorded point, and uses type-curve matching technique.

Matching of data on type curve is based on the relationship of the form

$$P_D(t_D) = \frac{2\pi k h \left(p_I - P_{wf}(t)\right)}{q\mu B} \tag{1b}$$

$$t_D = \frac{0.0002637kt}{\phi \mu C_t r_w^2} \tag{2}$$

Taking the logarithm of equation (1) and (2) provides th relation to be expected.

$$\ln(P_D(t_D)) = \ln\left(\frac{2\pi kh}{q\mu B}\right) + \ln(\Delta P(t)) \tag{3}$$

$$ln(t_D) = ln\left(\frac{0.0002637k}{\phi \mu c_t r_w^2}\right) + ln(t)$$
(4)

Log of P_D is a displacement of log of Δp . The dimensionless pressure P_D is plotted on the ordinate versus dimensionless time t_D on the abscissa and each curve corresponds to a specific values of skin, s and dimensionless wellbore storage C_D given by

$$C_D = \frac{0.8936C}{\phi c_t h r_w^2} \tag{5}$$

These curves are constructed from the analytical solution to diffusivity equation for constant terminal rate in a finite radius well with skin in an infinite reservoir. This satisfies the goals of transient testing which is to determine well and reservoir properties in the well drainage area with the aim of predicting the well performance. The pressure transient response can take have several particular flow regime in form of early radial flow, early linear flow, bilinear flow, pseudoradial flow and late radial flow as well as identifying faults boundaries.

For bounded reservoir system the use of type-curve is limited and hence with the introduction of pressure derivative in 1983 by Boudet, et al (1983) the various flow regimes in the reservoir limit test can be visually identified and by the virtue that inverse problems are solve by applicable regression models. The log-log plot of pressure and pressure derivative vs. time accompanied by the development of complex interpretation models that is appropriate to describe detailed geological features has put well testing as a powerful tool for reservoir characterization. Many of these geological features exhibit a unique signature on the derivative and can be inferred from analytical or numerically developed models, for examples in double porosity reservoirs and hydraulic fractured wells.

As illustration the dimensionless pressure for the late time in a bounded reservoir by van Everdingen and Hurst (1949) is given by

$$P_D(t_D) = \frac{2t_D}{r_D^2} + \ln(r_{DE}) - \frac{3}{4} \tag{6}$$

Taking the derivative of this equation gives

$$t_D \frac{\partial P_D}{\partial t} = \frac{2t_D}{r^2_{DE}} \tag{7}$$

The late radial flow regime pressure equation for the pulsing well is

$$P_D(t_D) = \frac{1}{2}(\ln(t_D) + 0.809 + 2s) \tag{8}$$

$$t_D \frac{\partial P_D}{\partial t_D} = \frac{1}{2} \tag{9}$$

The early well bore storage regime is given by

$$P_D(t_D) = \frac{t_D}{c_D} \tag{10}$$

$$t_D \frac{\partial P_D}{\partial t} = \frac{t_D}{C_D} \tag{11}$$

DEVELOPMENT OF REGRESSION EQUATION

In this section, we presented the baseline information for regression. Using vector space optimization approach for linear combination of set of measured data where for every time, a pressure value is measured and hence obtain

$$P_1 \atop P_2 \atop \cdot = \begin{pmatrix} 1 & ln(t_1) \\ 1 & ln(t_2) \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & t_n \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \leftrightarrow P = AT$$
(12)

To make the matrix a square matrix, we define a transpose T^t , such that T^tT is a square $2x^2$ matrix, thus

$$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ ln(t_1)ln(t_2)\cdots ln(t_n) \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ p_n \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ ln(t_1)ln(t_2)\cdots ln(t_n) \end{pmatrix} \begin{pmatrix} 1 & ln(t_1) \\ 1 & ln(t_2) \\ \vdots & \vdots \\ 1 & t_n \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \leftrightarrow P = AT \ (13)$$

and the multiplications produce

$$\begin{pmatrix}
\sum_{1}^{n} P_{i} \\
\sum_{1}^{n} P_{i} ln(t_{i})
\end{pmatrix} = \begin{pmatrix}
n & \sum_{i=1}^{n} ln(t_{i}) \\
\sum_{1}^{n} ln(t_{i}) & \sum_{1}^{n} \left(ln(t_{i})\right)^{2}
\end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$
(14)

Equation 14 can be solved for the variables a, b reducing the matrix to upper triangular form

$$\binom{\sum_{1}^{n} P_{i}}{\sum_{1}^{n} P_{i} \ln(t_{i}) - \frac{\sum_{1}^{n} P_{i} \sum_{1}^{n} \ln(t_{i})}{n}} = \binom{n}{0} \frac{\sum_{i=1}^{n} \ln(t_{i})}{\sum_{1}^{n} \left(\ln(t_{i})\right)^{2} - \frac{\left(\sum_{1}^{n} \ln(t_{i})\right)^{2}}{n}} \binom{a}{b}$$
 (15)

by back substitution we obtain

$$b = \frac{\sum_{1}^{n} P_{i} \ln(t_{i}) - \frac{\sum_{1}^{n} P_{i} \sum_{1}^{n} \ln(t_{i})}{n}}{\sum_{1}^{n} (\ln(t_{i}))^{2} - \frac{(\sum_{1}^{n} \ln(t_{i}))^{2}}{n}} = \frac{E(P \ln t) - E(P)E(\ln t)}{E((\ln(t))^{2}) - (E(\ln(t)))^{2}}$$
(16)

$$a = \frac{\sum_{i=1}^{n} P_i - b \sum_{i=1}^{n} ln(t_i)}{n} = E(P) - bE(ln(t))$$

$$(17)$$

Equations 16 and 17 determine the coefficient of correlation, which is the numerical measures of correlation and is given by

$$r = \frac{cov(P,t)}{\sqrt{var(P)var(t)}} \tag{18}$$

Where covariance is a measure of relationship between variables, and for linearly independent variables there covariance is zero implies their correlation coefficient is zero. Corter and Omolehin (2016) used variant of this model to build a predictive model for the two possible outcomes of cancer patients and for examination of which type of cancer is more deadly. Here we are using regression to predict reservoir transmissibility, storativity and the needs for stimulation.

The procedure developed so far are used in fixing measured data to identified models and the correlation coefficient, r is a measured of linearity of relationship between measured parameters

Interpretation Procedures

We discuss the interpretation model diagnostic plot effectiveness for pattern identification by the following field examples.

Results and Discussions

We discuss the interpretation model diagnostic plot effectiveness for pattern identification by the following a real field example. A drawdown test conducted in a finite reservoir system with the aims of establishing reservoir geometries and degree of damage is to be interpreted.

Table 1 show the reservoir and field data for an XY field whose geometrical configuration is to be confirmed from the flow pressure data in Table 2. The well was on a stabilized production rate of 250 Stb/day of oil with the initial pressure of 4412 Psia. The analysis of these data commences with plotting of both derivative and pressure and measure time on log-log plot shown in Figure 1. The test is interpretable by observation of the derivative plot with plateau region indicating the response from the reservoir away from the well bore. The three flow regimes identified are early radial flow with unit slope on the pressure curve, radial mid-term regime shown plateau signature on the derivative and pseudo-steady state of linear increase with time on the derivative. By this visual identification, appropriate regressions were deployed to obtain the result shown in Table 2. The derivative plot indicates that the interpretation models are such that the reservoir is one in which the well response data is from a well affected by wellbore storage and skin in a homogeneous reservoir with a closed outer boundary. This is indicated by the upward trend of the last few data on the derivative curve. The derivative data

are construcgle-plot procedureted by differentiation of actual measured data and we apply the Langrange interpolation formula to generate the data in table 1b. It should be noted that the derivative approach does not produce errors or noise but only reveal them as Figure 5 shows for transient test on another data. The regression equations for storage, mid-term and pseudosteady state regions are the following:

$$ln(P_{wf}(t)) = 6.5944 + 0.7764ln(t)$$
(19)

$$\Delta P_{wf}(t) = 757 + 31.171 ln(t) \tag{20}$$

$$\Delta P_{wf}(t) = 882.25 + 0.2185t \tag{21}$$

Equation 19 fits the early time data as linear on log-log plot and is useful to generate the wellbore storage coefficient, C. Its derivative is given as important parameter

$$\frac{dP_{wf}}{dt} = \frac{\Delta P_{wf}}{t} = \frac{0.7764t^{0.7764}e^{6.5944}}{t}, \text{ such that the wellbore storage coefficient is calculated from } C = \frac{qB}{24\frac{dp}{dt}} = 0.012979. \text{ The dimensionless storage coefficient is obtain from eqn. 5 as } CD = \frac{0.8936*0.012979}{0.039*1.7*10^{-5}*69*3^{\circ}2} = 6465.7$$

The mid-term region regression equation is therefore given by eqn. 21 and compared to eqn. 5, we have the slope $m=31.179=\frac{70.64\mu B}{kh}$. We solve for reservoir transmissibility $\frac{kh}{\mu}=\frac{70.6*250*1.136}{31.171}=642.634~MD-ft/cp$ and given permeability k=7.456~MD.

Further for clarifications, the regression procedure for mid-term region is shown in Table 3. The computer sequential control step procedure is shown in Table 4 and the summary of interpreted results is shown in Table 5

CONCLUSIONS AND RECOMMENDATIONS

The paper presented the filtering capacity of derivative plot to identify preliminary well test interpretation models and hence describe reservoir and well geometries. The whole performance estimation is done by convectional methods. The approach provides effective and unbiased way to examine and understand welltest results. Regression equation is defined here in conjunctions with the minimization in the squared mean deviation and functions as bringing a representation equation to stand for the identified model. This can be viewed in the case of pseudo-steady, the dimensional equation 6 has either of these representations as propagated by Ozka and Raghavan (1991) and Ramey (1992)

$$\Delta P = P_i - P_{wf} = \frac{0.077447qB}{\emptyset C_t h r_e^2} t + \frac{70.6q\mu B}{kh} \left(ln \left(\frac{4A}{\gamma C_A r_w^2} \right) + 2s \right)$$
 (23)

$$\Delta P = \frac{0.23395qB}{\emptyset C_t hA} t + \frac{70.6q\mu}{kh} \left(ln \left(\frac{4A}{\gamma C_A r_W^2} \right) + 2s \right)$$

$$\tag{24}$$

Equation 24 will be in general written in the form $\Delta P = a + bt$. Equation 4 was failed to be developed by Erhunmwun and Akpobi (2016) using finite element scheme. The slope calculates both the drainage area and reservoir pore volume in communication with the well bore. The intercept will be the measure of depletion or production given by

$$\Delta P = P_i - P_{wf} = \frac{70.6q\mu B}{kh} \left(ln \left(\frac{4A}{\gamma C_A r_w^2} \right) + 2s \right)$$
 (25)

Equation 25 is useful for reservoir description as it provides information on the reservoir configuration and location of the well within the configuration. Erhunmwun (2016) presentation still did not identify this in their development which seemly makes their derivation not useful for reservoir description.

This approach is hereby recommended as well test expert system and pseudo-computer aided welltest interpretation. The regressions are to be performed on specific recognized area with the understanding of patterns of geometric reservoir architecture and wellbore geometry.

Thus, quality of well test interpretations are improved considerably by the availability of accurate pressre data and deployment of software for computer aided analysis. The use of derivative pressure vs. time is mathematically justified because it is included in one term the diffusivity equation. It has become global approach to diagnose the pressure behavior and to identify the various characteristic flow regimes and, in addition, specialized analysis, valid for specific flow regimes, are performed on selected portions of pressure data. As final conclusions, the derivative approach improves the definition of analysed plots and therefore the quality of interpretation. The interpretation of pressure is a single-plot procedure. If enough data are available, pressure and time matches are fixed, so the analysis is faster. Quick decisions during test saves rig time.

NOMENCLATURE

A = Drainage Area, Sq. Ft.

B = Formation Volume Factor, RB/STB

 C_t =Total Compressibility, psi^{-1}

P =Pressure, psi

 ΔP = Pressure drawdown, Psia

 $t \frac{\partial P}{\partial t}$ = Pressure Derivative, Psi

 $c_t = total compressibility, psi^{-1} (Pa^{-1})$

 η = Coefficient of diffusion

h = Thickness, ft

K= formation permeability, MD.

μ= Fluid viscosity

 ΔP = pressure drawdown, psia

 P_D = dimensionless pressure

t = time, hrs, min.

 $t_D = dimensionless time$

 ϕ = Porosity

 γ = exponential parameter of Euler number approx. 0.5772

s = skin factor, dimensionless

 C_D = dimensionless wellbore sorage coefficient

C = wellbore sorage coefficient, B/D

EI (x) = exponential integral solution, - EI(-x) =
$$\int_{x}^{\infty} \frac{e^{-U}}{u}$$

Cov() = covariance of two variables

R =coefficient of correlation

 ΔP_o = Pressure drop at the start of pseudosteady state.

Subscripts

D = dimensionless

i = Initial

o = onset

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APPENDIX

Table 1: well fluid and rock properties

q, stb/day	250	Bo RBl/stb	1.136	visc, Cp	0.8	por =	0.039
ct 1/ psia=	1.70E- 05	h ft	69	Pi Psia	4412	rw ft	0.198

Table 2: Measured field Pressure-Time Data

time,	Pwf,	time,	Pwf,	time,	Pwf,	time,	Pwf,
hrs	Psia	hrs	Psia	hrs	Psia	hrs	Psia
0.12	4272	1.94	3699	24.9	3555	154	3497
0.2	4201	2.79	3653	29.8	3549	185	3490
0.3	4122	4.01	3636	35.8	3544	222	3481
0.4	4057	4.82	3616	43	3537	266	3472
0.5	3995	5.78	3607	51.5	3532	319	3460
0.6	3953	6.94	3600	61.8	3526	383	3446
0.7	3912	8.32	3593	74.2	3521	460	3429
0.8	3884	14.4	3573	89.1	3515		
0.9	3857	17.3	3567	107	3509		
1	3822	20.7	3561	128	3503		

Table 3: Regression result for Mid-term Transient region.

Num	T	Ln(t)	р	t^2	t*P	p^2
1	29.8	3.394508	3549	11.52269	12047.11	12595401
2	35.8	3.577948	3544	12.80171	12680.25	12559936
3	43	3.7612	3537	14.14663	13303.36	12510369
4	51.5	3.941582	3532	15.53607	13921.67	12475024
5	61.8	4.123903	3526	17.00658	14540.88	12432676
6	74.2	4.306764	3521	18.54822	15164.12	12397441
7	89.1	4.489759	3515	20.15794	15781.5	12355225
SUM =	385.2	27.59567	24724	109.7198	97438.89	87326072
Mean	55.02857	3.942238	3532	15.67426	13919.84	12475153

Denom	-202.97	nume=	6.51806	b=	-31.1396	a=	3654.76
var(t)=	0.133022	var(p)=	129.1429	cov=	-4.14225	r=	0.9994

Table 1b: Pressure and Pressure Derivative Data

Time	Dp	Derv 1	Derv 2	Time	Dp	Derv 1	Derv 2	Time	Dp	Derv 1	Derv 2
0.12	140			4.82	796	109.01	78.972	74.2	891	27.419	29.897
0.2	211	142	166.67	5.78	805	49.688	43.623	89.1	897	32.879	32.598
0.3	290	197.5	216	6.94	812	38.379	38.252	107	903	32.866	33.008
0.4	355	227.5	254	8.32	819	38.703	30.113	128	909	33.571	32.681
0.5	417	279	260	14.4	839	37.368	41.693	154	915	32.538	35.123
0.6	459	231	249	17.3	845	32.793	32.952	185	922	38.274	43.529
0.7	500	266.5	241.5	20.7	851	33.529	32.684	222	931	49.5	49.333
0.8	528	210	220	24.9	857	32.571	32.835	266	940	49.909	57.588
0.9	555	229.5	279	29.8	863	33.49	30.073	319	952	66.226	70.889
1	590	332.5	151.92	35.8	868	27.333	32.545	383	966	76.781	84.206
1.94	713	192.35	183.16	43	875	38.306	32.866				
2.79	759	127.99	84.913	51.5	880	27.794	30.133				
4.01	776	47.377	73.089	61.8	886	33	29.947				

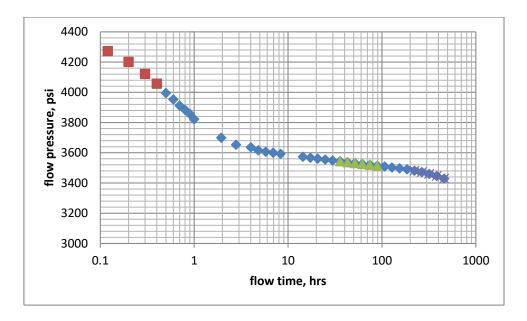


Figure 1: Semilog graph of Pressure-Time Data

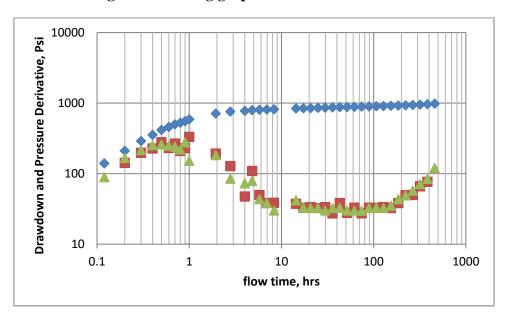


Figure 2: Pressure and Derivative Curves

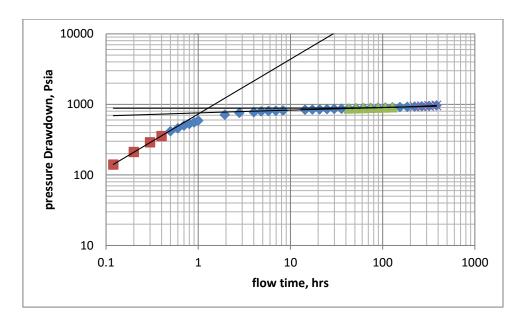


Figure 3: Log-Log Plot of Pressure Drawdown vs. Time

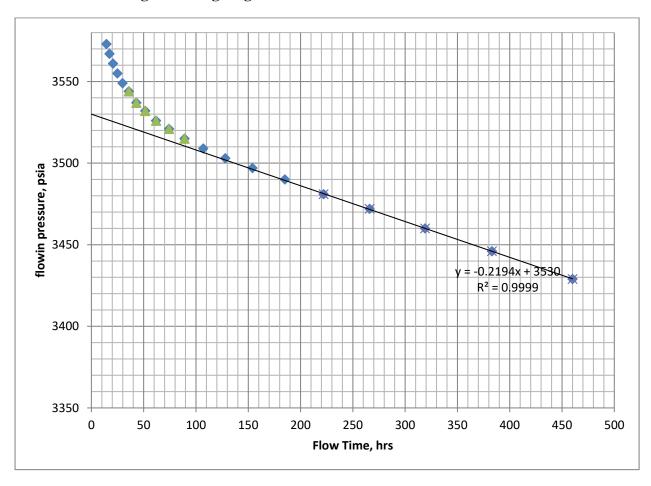


Figure 4: Linear plot for estimation of drainage area and configuration

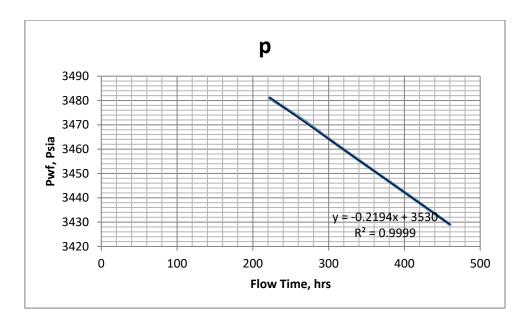


Figure 5: Abridged Version of Table 4 Describing Late Response

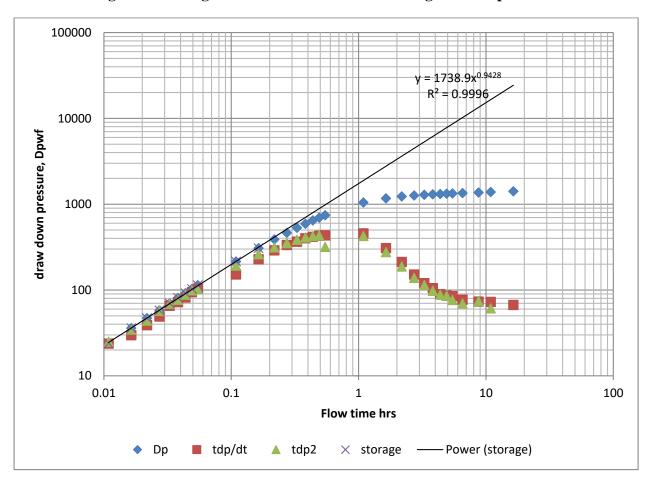


Figure 5: Pressure and Pressure Derivative plot with less Noise for Transient pressure test

Table4: Computer print out of general solution

SLOPE									
m=	31.2	p inter=	3655	m-log=	71.854	line slope	0.2194	po =	3530
								Pi-	
kh =	514.11	k MD =	7.4508	Trw^2=	2E-08	St	4E+08	P1hr=	757
		Dpskin,							
s=	6.0393	psi	377.54	PI ideal=	0.3752	Actual PI=	0.7484	FE =	0.5013
early		EAR							
slope	0.7764	INT	730.97	DP/DT=	911.76	C=	0.013	CD =	6465.2
		twb hrs		pore vol. Ft^3		pore vol.		Area,	
kh/visc=T	642.63	=	5.5028	=	2E+07	Bls. =	3E+06	ft^2=	7E+06
acres=	151.94	pi-po =	882	4A/(1.781rw^2	4E+08	log(CA) =	1.5581	CA =	32.5
ln(CA)=	3.5629	CA =	33						

Table 4: Summary of Results

				A,	
k, md	S	C Bl/Psia	CD	acres	CA
7.456	6.04	0.012979	6143.7	152.6	32.1